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MOMENT-ROTATION CHARACTERISTICS OF COLUMN ANCHORAGES

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SYNOPSIS

This paper presents methods of obtaining the moment-rotation characteristics for common types of light industrial building column anchorages. These characteristics are useful in methods of analysis or design, particularly where resistance to rotation may be critical to the survival of the structure under heavy lateral loads such as in earthquake or blast loads. Upper and lower bounds for maximum resisting moment and maximum rotation, respectively, are developed. Shear is found to have little effect on the ultimate resisting moment but has somewhat more effect on the maximum rotation.

Formulation of the complete moment-rotation curve is developed in five stages. Appendices illustrate the use of the formulae. The variables involved are discussed and the effect of each on the ultimate moment and rotation is shown graphically. The procedure developed may provide at least a first approximation until experimental data become available allowing corroboration or correction of the assumptions that have been made.

INTRODUCTION

To permit the analysis of a complete structure the knowledge of the ultimate strength of the column anchorages must be supplemented by an estimate of their moment-rotation characteristics, particularly so in the case of structures subjected to lateral blast loads.

Since analytical and test information on the ultimate behavior of column anchorages is largely lacking, a theoretical analysis of the problem applicabie to simple types of anchorages is developed.

The degree to which anchorages affect the capacity of structures to resist lateral loads, such as could occur as a result of a bomb blast, varies with the type of structure. This follows from consideration of the three ways in which a column anchorage can fail:

1) failure in shear resistance
2) failure in moment resistance, and
3) failure in tensile resistance.

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Shear failure of an anchorage might take place in a low, wide building. This failure could take the form of cracking of the concrete pier to which the anchorage is secured or shearing off of the anchor bolts. The latter type of failure might occur due to shear alone or in combination with the applied moment, causing combined tension and shear failure of individual bolts. Anchorages of multistory buildings are unlikely to fail in shear in view of their high degree of rigidity and the great amount of friction due to vertical loads. While each design should be examined individually, in general the anchorages of tall buildings, being heavily embedded in concrete, can be considered fully fixed; hence, deformation and failure would be governed by the properties of the columns.

A related problem outside the scope of this paper is the possibility of overall footing failure. Between the two extreme cases of fixed and flexible anchorages is the group encountered in most light industrial buildings. These anchorages consist of plates attached to the bottom of columns and held down by anchor bolts embedded in a concrete pier (Fig. 1). The moment-rotation relationship of this type of anchorage will be investigated omitting consideration of any relative movement between the concrete foundation and the surrounding soil.

A number of technical articles have appeared on the subject of the design of simple anchorages,(6) but a review of literature has not revealed any experimental or analytical investigation of such anchorages. Empirical procedures have been used for design purposes with the knowledge that a large factor of safety is available to absorb discrepancies between the assumptions and the actual behavior of anchorages. The assumptions made herein are more realistic and are readily adjustable to results of tests as they become available.

Upper Bound[4] of Moment

Rectangular Base Plate

The most important variables that enter into the moment-rotation relationship of column anchorages are:

i) dimensions of base plate;
ii) dimensions, location, and stress-strain characteristics of anchor bolts;
iii) dimensions and stress-strain characteristics of concrete pier; and
iv) vertical loading.

As a first step in determining the manner in which changes in these variables affect the moment-rotation relationship of the anchorage, the upper bound of the moment and the corresponding minimum rotation will be estimated.

It is convenient to express many variables in terms of the dimensions of the base plate; thus, all primed symbols signify such a manner of expression. For example, e is defined as the distance of an anchor bolt from the center line of the base plate and $e' = e/(1/2\ d) = 2e/d$. The only exceptions to this use of primed symbols is f'_c, which has the conventional meaning of ultimate concrete (cylinder) compressive strength.

4. "Upper bound" means an approximate evaluation of a resisting moment below which the actual resting moment must lie.

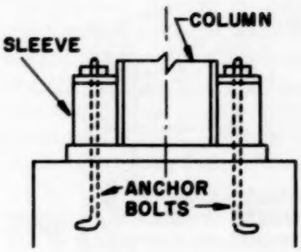


Fig. 1.

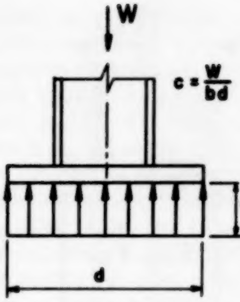


Fig. 2

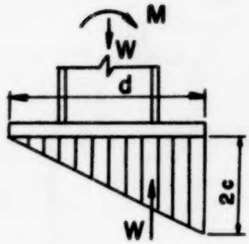


Fig. 3.

Upper Bound for Moment

Consider a column anchorage without anchor bolts as shown in Fig. 2. The unit bearing pressure "c" due to a concentric vertical load W is assumed to be uniform. On a concrete foundation the maximum value for c permitted by most codes is 600 or 800 psi. If a moment is now applied by the column, the stress distribution at some stage will be assumed to be approximately as shown in Fig. 3. Evidently, the resisting moment is W d/6, where d is the length of the base plate. If, hypothetically, the concrete had infinite strength and the bearing plate were infinitely rigid, then equilibrium considerations alone would determine the maximum moment that could be resisted. Just before the column toppled over, as shown in Fig. 4, the resultant base pressure would be at the edge of the plate and the "upper bound" moment would be

$$M_u = \frac{1}{2} Wd. \tag{1}$$

Next, consider the same column and base plate arrangement supporting no weight at all, but this time held down by anchor bolts. The plate and the concrete are again assumed as infinitely rigid. The degree to which the bolts have been tightened is immaterial. Let the ultimate strength of the bolt or bolts on either side of the center line of the base plate be P_u. Also, let the distance from the anchor bolts to the center line be e, as shown in Fig. 5. In this case the upper bound for the resisting moment is

$$\begin{aligned} M_u &= P_u \left(\frac{d}{2} + e\right) + P_u \left(\frac{d}{2} - e\right) \\ &= 2P_u \frac{d}{2} \end{aligned} \tag{2}$$

Adding the upper bound moments for the two cases (Equations 1 and 2), the upper bound for moment for an anchorage supporting a weight W and held down by anchor bolts of ultimate strength $2P_u$ is

$$M_u = (W + 2P_u) \frac{d}{2} \tag{3}$$

If interested only in the moment which without any doubt would cause complete structural failure of the type of anchorage under consideration, Equation 3 would provide sufficient information.

The symbol p is now introduced for A_t/bd, where A_t is the cross-sectional

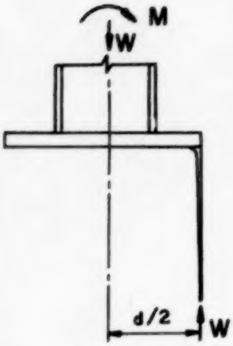


Fig. 4.

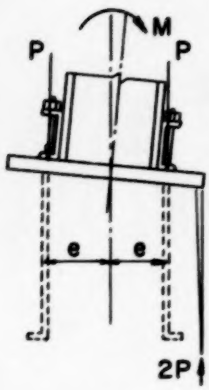


Fig. 5.

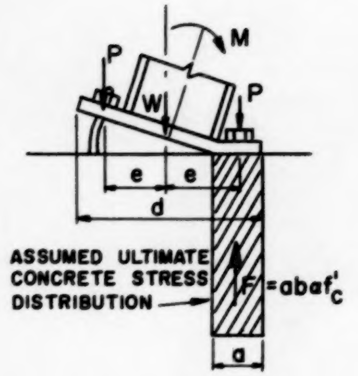


Fig. 6.

area of half the anchor bolts and b is the width of the base plate. Thus p has a meaning similar to that in the reinforced concrete theory.[5] Equation 3 can thus be rewritten

$$M_u = (c\ bd + 2p\ \sigma_u\ bd)\ \frac{d}{2},$$

where σ_u = ultimate steel strength.

Thus, $M_u = (c + 2p\ \sigma_u)\ \frac{bd^2}{2}$.

Writing

$$\frac{M_u}{bd^2} = M'_u$$

$$M'_u = \frac{1}{2}\ (c + 2p\ \sigma_u)\ .$$

Hence, the upper bound moment divided by bd^2 is the same for similar anchorages of different size, provided that the original uniform bearing stress and the ratio of anchor bolt area to base plate area remains the same.

Next, a more realistic assumption is made with regard to the concrete strength. The actual ultimate concrete strength depends to some degree on the relative size of the base plate and the concrete footing. It is known that the compressive strength of concrete confined in one or more directions is greater than the value of f'_c determined from standard cylinder tests. It will be assumed, therefore, that the concrete strength is $\alpha f'_c$, where $\alpha \geqq 1$ and must be either estimated or determined by suitable tests.

Theoretical considerations based on elastic theory(1) suggest that α might be between 1 and about 1.5. Tests carried out by the Hydro-Electric Power Commission of Ontario (unpublished) on concrete cubes loaded over a small central square indicate that α may be proportional to the cube root of the ratio of total area to loaded area. Values for α as high as 10 have been obtained in tests at the Commission's laboratories.

5. In reinforced concrete theory "d" is the distance from the extreme compression fibers to the reinforcing steel.

From the viewpoint of determining the moment resistance of anchorages, the effect of loading a rectangular strip is of more interest than that of loading a small square. Graf[2] reports on tests performed on concrete cubes in which ultimate strength under a loading on a strip covering only a third of the top surface was 1.6 times as high as the ultimate strength of a cube loaded uniformly over all its cross-section. Loading on a strip one-seventh of the width of the cube resulted in values for α varying from 2.5 to 2.8. Moving the loaded strip toward the edge of the cube resulted in a reduction of α to 1.2 to 1.5. Fortunately, as is shown later, the values M_u are relatively insensitive to changes in α. The concrete stress distribution just before failure is likely to be nearly trapezoidal, but it is considered that in view of the large strains involved, no great error is introduced by assuming the stress distribution to be uniform, as shown in Fig. 6. Let the width of the strip of concrete in touch with the base plate be a. Then let

$$\alpha f'_c ab = W + P_u, \text{ for } \left(\frac{d}{2} - e\right) < a \tag{4a}$$

or

$$\alpha f'_c ab = W + 2P_u, \text{ for } \left(\frac{d}{2} - e\right) > a . \tag{4b}$$

Analyzing first the case when $(d/2 - e) < a$, i.e., the bolts near the compression zone are not effective, and defining a_1 as the width of concrete bearing area in this particular case,

$$a_1 = \frac{W + P_u}{\alpha f'_c b} = \frac{(c + p\,\sigma_u)d}{\alpha f'_c}$$

$$M_{u1} = W\left(\frac{d}{2} - \frac{a_1}{2}\right) + P_u\left(\frac{d}{2} + e - \frac{a_1}{2}\right)$$

Writing

$$e' = \frac{e}{(1/2)\,d} = \frac{2e}{d}$$

$$M_{u1} = \frac{cbd^2}{2}\left[1 - \frac{c + p\,\sigma_u}{\alpha f'_c}\right] + \frac{p\,\sigma_u\,bd^2}{2}\left[1 + e' - \frac{c + p\,\sigma_u}{\alpha f'_c}\right],$$

$$M'_{u1} = \frac{M_{u1}}{bd^2} = \frac{1}{2}(c + p\,\sigma_u)\left(1 - \frac{c + p\,\sigma_u}{\alpha f'_c}\right) + \frac{1}{2}p\,\sigma_u\,e' \tag{5a}$$

For the second case, when

$$(\frac{d}{2} - e) > a$$

$$a_2 = \frac{W + 2P_u}{\alpha f'_c b} = \frac{(c + 2p \sigma_u)d}{\alpha f'_c}$$

then

$$M_{u2} = W(\frac{d}{2} - \frac{a_2}{2}) + P_u (\frac{d}{2} + e - \frac{a_2}{2}) + P_u (\frac{d}{2} - e - \frac{a_2}{2})$$

$$= W(\frac{d}{2} - \frac{a_2}{2}) + 2P_u (\frac{d}{2} - \frac{a_2}{2})$$

$$M'_{u2} = \frac{1}{2} (c + 2p \sigma_u) (1 - \frac{c + 2p \sigma_u}{\alpha f'_c}) , \tag{5b}$$

which is seen to be independent of e.

M'_{u1} and M'_{u2} are shown in Fig. 7, plotted against e' for specific values of c, p, $\alpha f'_c$, and σ_u. The larger one of the two is the upper bound.

In most cases the value of e' does not differ much from 0.8 and the effective value of the upper bound will be in the range where M'_{u1} is greater than M'_{u2}. In the following, the expression for M'_{u1} will be used for M'_u with rectangular base plates.

A study of Equations 5a and 5b reveals that in any practical case all variables except $\alpha f'_c$ are known. To illustrate the effect on M'_{u1} of any error in estimating $\alpha f'_c$ the graph in Fig. 8 was prepared, again using specific typical values of c, p, e', and σu. M'_{u1} is greater than M'_{u2} up to values of $\alpha f'_c$ of about 11000 psi which is near the limit of practicality. Hence, M'_{u1} is the effective upper bound all the way. The change in M'_{u1} for a change in f'_c from 4000 to 8000 psi is only 7 percent. Even if a considerable error is made in estimating $\alpha f'_c$, a fairly accurate estimate of M_u should be possible.

No assumption has been made in the above as to the manner of application of the moment because such an assumption is not required. The moment which is considered here is the total external moment and in the case of large lateral displacements of the structure, vertical loads may be responsible for a considerable fraction of the total moment.

Lower Bound for Rotation

The amount of rotation associated with the ultimate moment cannot be estimated with any degree of accuracy, since it depends to a large extent on the elongation of the anchor bolts. The bolt elongation is more variable than that of plain rods and although the elongation of a tensile specimen made of the

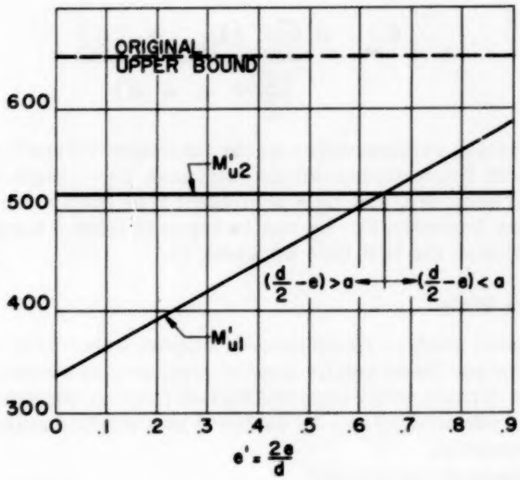
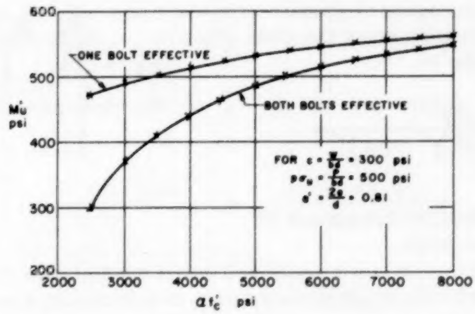


Fig. 7.
Upper Bound for Moment: Variation With Eccentricity Using the Following Typical Values; $\alpha f'_c$ = 6000 psi, c = W/bd = 300 psi, $p\sigma_u$ = P/bd = 500 psi.



$$M'_{u_1} = \frac{M_{u_1}}{bd^2} = \frac{1}{2}(c+p\sigma_u)\left(1-\frac{c+p\sigma_u}{\alpha f'_c}\right)+\frac{1}{2}e'p\sigma_u \quad \text{......... (5a)}$$

$$M'_{u_2} = \frac{M_{u_2}}{bd^2} = \frac{1}{2}(c+2p\sigma_u)\left(1-\frac{c+2p\sigma_u}{\alpha f'_c}\right) \quad \text{......... (5b)}$$

Fig. 8.
Upper Bound for Moment: Variation With Ultimate Concrete Strength.

same material may be 20 to 30 percent, the bolt may elongate no more than 5 percent at failure. However, it is possible to estimate a minimum value for the rotation based on the extension of the bolt at the maximum stress.

From Fig. 9 the lower bound for the rotation is

$$\theta_L = \frac{\epsilon_u (L_1 + L_2)}{(\frac{d}{2} + e - a)}$$

where ϵ_u = strain, corresponding to the maximum stress for the material of the anchor bolts (from stress-strain diagram); L_1 = length of bolt above the surface of the concrete; and L_2 = equivalent free length of embedded portion of the bolt (see Appendix 1). L_2 can be taken at least 8 times the diameter of the plain portion of the bolt (See Appendix 1).

Circular Base Plate

Pipe columns, such as those used to support water towers, rest on circular base plates and these can be treated similarly to rectangular plates. The case with two diametrically opposed anchor bolts is considered here, but other types of anchorages can be analyzed in a similar manner to find the upper bound moment.

As in the case of rectangular plates, it is assumed that at the ultimate moment only part of the base plate is in contact with the concrete (Fig. 10) and that the bearing pressure there is uniform and equal to $\alpha f'_c$. The distance to the center of gravity of the compression area from the center of the base plate is

$$z = \frac{d \sin^3 \gamma}{3 (\gamma - \sin \gamma \cos \gamma)} .$$

With one bolt ineffective because it is in the compression zone,

$$M_{u1} = Wz + P_u(z+e) = Wz + A_s \sigma_u (z+e) .$$

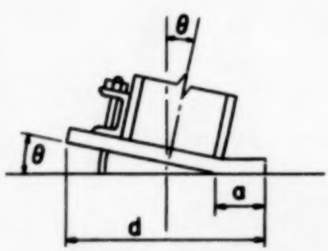


Fig. 9.

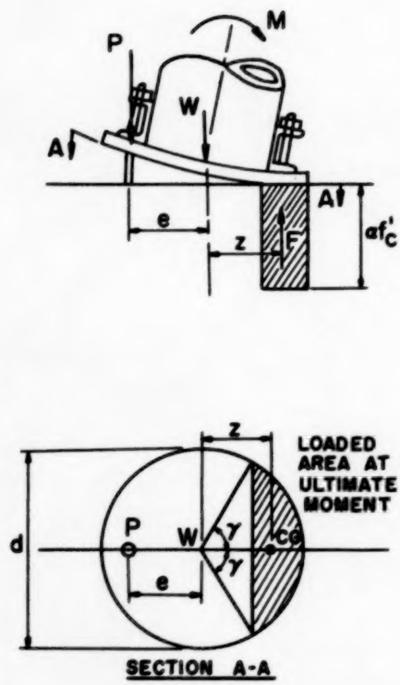


Fig. 10.

Let,

$$M_{u1} = \frac{M_{u1}}{\frac{\pi}{4} d^3}$$

$$c = \frac{W}{\frac{\pi}{4} d^2}$$

$$p = \frac{A_t}{\frac{\pi}{4} d^2}$$

$$e' = \frac{e}{\frac{1}{2} d}$$

$$M'_{u1} = \frac{(c + p\ \sigma_u) \sin^3 \gamma}{3(\gamma - \sin \gamma \cos \gamma)} + \frac{p\ \sigma_u\ e'}{2} \quad , \qquad (6)$$

which can also be written

$$M'_{u1} = \frac{\alpha f'_c \sin^3 \gamma}{3} + \frac{P\ \sigma_u\ e'}{2} \quad .$$

Furthermore, since at the stage considered the compression force in the concrete must equal the ultimate strength of one bolt plus the load W,

$$\left(\frac{d^2}{4}\gamma - \frac{d^2}{4} \sin \gamma \cos \gamma\right) \alpha f'_c = p\pi \frac{d^2}{4} \sigma_u + c\pi \frac{d^2}{4}$$

$$\alpha f'_c = \frac{\pi\ (p\ \sigma_u + c)}{\gamma - \sin \gamma \cos \gamma} \quad . \qquad (7)$$

It is difficult to eliminate γ from Equations 6 and 7 but they can be used together with γ as a parameter in the same way as Equation 5 is used alone. For example, a curve relating M'_{u1} to $\alpha f'_c$ can be plotted (Fig. 11). Comparing Figs. 11 and 8 it is seen that for the same values of $\alpha f'_c$, M'_{u1} is slightly smaller for a circular base plate. It should be recalled, however, that the factor by which M'_{u1} is multiplied in the two cases to obtain the upper bound moment is not the same.

FOR A CIRCULAR BASE PLATE WITH TWO ANCHOR BOLTS

$$M'_{u1} = \frac{\sin^3 \gamma}{3(\gamma - \sin \gamma \cos \gamma)} (c + p\sigma_u) + \frac{p\sigma_u e'}{2} \qquad (6)$$

$$\alpha f' = \frac{\pi(p\sigma_u + c)}{\gamma - \sin \gamma \cos \gamma} \qquad \ldots (7)$$

WHERE γ IS THE ANGLE SHOWN IN FIG 10

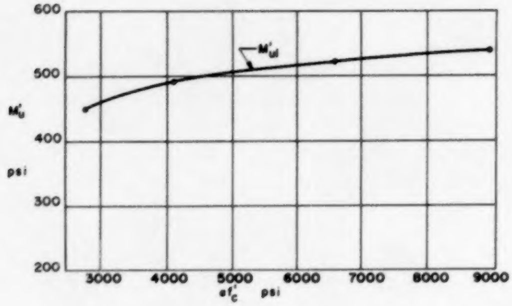


Fig. 11.

Variation of Upper Bound Moment With Ultimate Concrete Strength For a Circular Base Plate

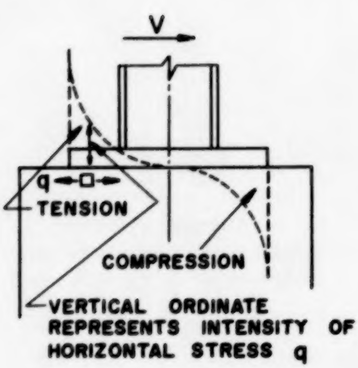


Fig. 12.

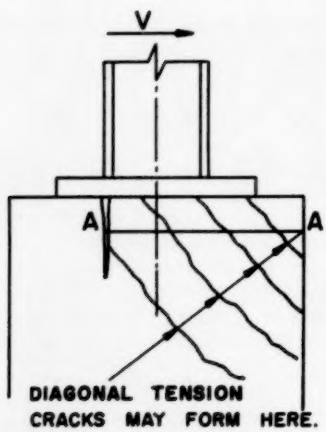


Fig. 13.

Effect of Shear

Horizontal shear on a simple anchorage is resisted by the anchor bolts and by friction. The resistance of the anchor bolts is, of course, the shear strength multiplied by the cross-sectional area.

The horizontal resistance due to friction is μW. When the base plate has rotated so as to cause its toe to dig into the concrete, larger horizontal forces might be transmitted.

In analyzing the effect of the friction forces alone (assuming uniform bearing pressure) it is found that they produce compression in a horizontal direction under one-half of the base plate and tension under the other half. The variation of these stresses is shown in Fig. 12, and, theoretically, infinite stresses exist near the edges. Actually, plastic deformation reduces the peaks and, furthermore, when cracks on the tension side reduce the rigidity of the concrete on that side most or all the resistance is provided on the compression side. When the toe of the base plate digs into the concrete, all the resistance is concentrated along one line. Tracing further the path of the horizontal forces it is found that unless the footing is completely embedded in fairly rigid material, the forces have to be transmitted to lower parts of the footing through shear along sections such as AA in Fig. 13. When the horizontal load is concentrated at the toe of the base plate, the distance AA is quite short and the high diagonal tension may result in the corner of the footing spalling off. Failure due to excessive compression could, however, be evidenced by similar symptoms and the two should not be confused. The conclusion to be drawn from this discussion is that not all the horizontal resistance which appears to be available as a result of friction between base plate and concrete may be effective, owing to the possibility of premature failure in another mode.

The Effect of Shear on the Ultimate Moment

The effect of combined tension and shear on the strength of a rivet was studied by H. L. Cox,[7] with correlation to test results. He pointed out that the relationship between the tensile and shear components and their resultant at failure can be represented by a quarter ellipse, as shown in Fig. 14. It is proposed to assume that a similar relationship holds good for the unthreaded portion of an anchor bolt.

The question arises, what fraction of the ultimate tensile strength of the anchor bolts is available to resist moment if shear forces are also acting on the bolts. If $r\sigma_u$ is this fraction when the shear stress on the anchor bolts is τ, then from Fig. 14 the relation between τ, r, and σ_u is

$$\tau = \frac{3\sigma_u}{4} \sqrt{1 - r^2}$$

Let the shear force causing this shear stress be V_s. Since this force is resisted by all the anchor bolts,

$$V_s = 2A_t\tau = 2p\ bd\ \tau .$$

Furthermore, let the additional shear force due to friction between the base plate and the concrete be $V_F = \mu F = \mu\ (W + P)$.[6] Hence, the total applied shear $V = V_S + V_F$. Set the ratio $M_u/V = h = h'd$, i.e., the distance from the base plate at which the shear V would have to be applied to result in a moment M_u.
Now

$$\tau = \frac{V_S}{2pbd} = \frac{V - V_F}{2pbd} = \frac{\frac{M_u}{h'd} - \mu(W + P_u)}{2pbd}$$

Dividing numerator and denominator by bd,

$$\tau = \frac{M_u'}{2ph'} - \frac{\mu\ (c + pr\sigma_u)}{2p} .$$

Equating the two expressions for τ,

$$\frac{3\sigma_u}{4}\sqrt{1 - r^2} = \frac{M_u'}{2ph'} - \frac{\mu c}{2p} - \frac{\mu r\sigma_u}{2} ,$$

and hence

$$h' = \frac{(c + pr\sigma_u)\left(1 - \frac{c + pr\sigma_u}{\alpha f_c'}\right) + pr\sigma_u e'}{3\ p\sigma_u\sqrt{1 - r^2} + 2\ \mu(c + pr\sigma_u)} . \qquad (8)$$

Equation 8 establishes a relationship between h' and the fraction r of the ultimate tensile strength that can be counted on at failure. The relationship is shown in graph form in Fig. 15 for some typical values of c, p, σ_u, $\alpha f_c'$, e', and μ. This shows that for values of $h > 1.38$ d, $r = 1$, i.e., no shear stress exists in the anchor bolts, implying that all the shear is resisted by friction.

Although the shape of the graph would change with different values of the various parameters, it is evident that only for very small values of h would the ultimate tensile strength be seriously reduced by the action of shear. As

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6. The full frictional force is used here for illustration, but its complete omission does not greatly affect the numerical results.

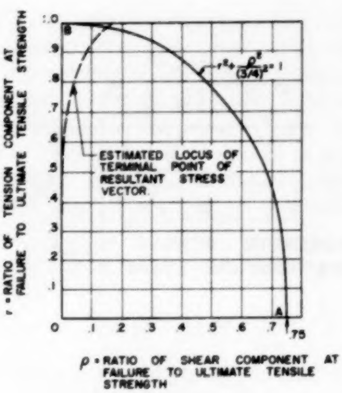

Fig. 14.
Effect of Combined Shear and Tension on Ultimate Strength of Round Steel Bar.

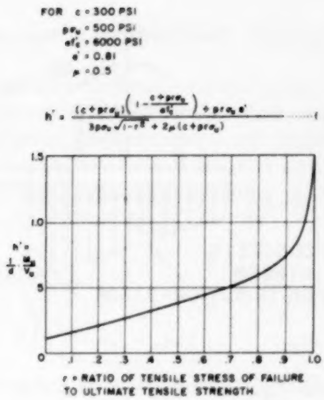

Fig. 15.
Effect of Moment-Shear Ratio On Ultimate Anchor Bolt Strength.

the anchorage yields and rotates, the value of h decreases and h will approach zero as the plastic hinge forms. This could change the relative magnitude of the shear and tensile components in a manner illustrated by the dotted line CC in Fig. 14. The point C determines the stress components at failure. Although free rotation may eventually take place it is unlikely that C will be below $r = 0.9$. In general, it is suggested that, in regard to the degree of accuracy to which the ultimate strength is known, the effect of shear on the ultimate strength can be neglected.

The General Moment-Rotation Relationship

The method described earlier of calculating the upper bound for the resisting moment of an anchorage, apart from being of interest in itself, helps to visualize the relation between the variables at all stages.

Let it be assumed that during the first stage of application of moment the concrete stress distribution is as shown in Fig. 16. If the highest stress is f_c, then the least stress must be $2c - f_c$, since the average stress is $W/bd = c$. At the end of stage 1, $2c - f_c = 0$, i.e., $f_c = 2_c$.

During the stage 2 the point of zero stress moves across from the edge of the plate toward the first anchor bolt. Stage 2 ends as the point of zero concrete stress passes the anchor bolt, i.e., when tensile stress is induced in the bolt.

Stage 3 is considered to end when yield point stress is first reached in the anchor bolts. The forces on the anchorage during stage 3 are shown in Fig. 17. Equilibrium considerations yield only one equation, namely

$$P + W = F ,$$

but this is not sufficient to permit the calculation of the concrete stress distribution diagram. Specifically, the distance w, shown in Fig. 17, is required to calculate values of M at any stage,

$$M = W(\frac{d}{2} - w) + P(\frac{d}{2} + e - w),$$

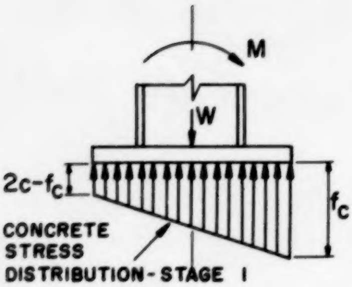

Fig. 16.

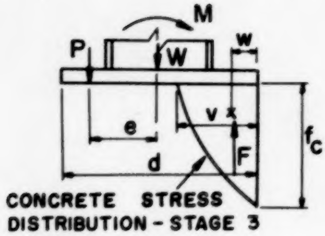

Fig. 17.

or if

$$w' = \frac{w}{\frac{d}{2}} = \frac{2w}{d}$$

$$M' = \frac{1}{2}\ (c + p\sigma)\ (1 - w') + \frac{1}{2}\ p\sigma\ e'. \qquad (9)$$

Stage 4 is considered to terminate when the anchor bolt strain reaches the strain hardening range. Expression 9 for M' still applies, but the value of w' is now different from that in stage 3. The remaining stage, up to failure, is called stage 5.

In a previous section of this paper the ultimate value of M' was estimated by assuming that the concrete stress just before failure would be uniformly distributed and equal to the ultimate concrete strength as modified by the prevailing conditions of restraint.

To estimate the value of θ, the angle of rotation of the column base, corresponding to a moment M and a steel stress σ the strains at the end of stage 1 are considered, i.e., when the concrete stress is zero under one edge of the base plate and 2c under the other edge. The difference in the displacement at the two edges can be approximately calculated in two ways. Assuming that the depth to which the stresses are transmitted is equal to width d and ignoring the effect of the concrete all around the base plate, the differential movement is $2cd/E_c$ and $\theta = 2c/E_c$. Alternatively, Timoshenko(1) gives an expression for the average deflection under a rectangular plate subject to a load F, namely,

$$F\ \frac{(1 - \nu^2)}{E_c\ \sqrt{bd}} \qquad \text{approximately, } (\nu = \text{Poisson's ratio})$$

$$= \frac{(c + p\sigma)\ bd\ (1 - \nu^2)}{E_c\ \sqrt{bd}} = \frac{(c + p\sigma)\ \sqrt{bd}\ (1 - \nu^2)}{E_c} .$$

Assuming that the maximum deflection is twice the average deflection,

$$\theta = \frac{2\ \sqrt{bd}\ (1 - \nu^2)}{E_c\ d}$$

and when numerical values are substituted, the result is practically the same as obtained from the first expression for θ.

The second stage is only of brief duration and can be treated similarly to stage 1. In the third, fourth, and fifth stages rotation is due to three causes: (i) extension of the anchor bolts; (ii) compression of the concrete; and (iii)flexure of part of the base plate. A knowledge of the anchor bolt extension and the distance v (see Fig. 17) would permit the calculation of θ for any value of σ and for any value of M', if w were also known. In this stage

$$\theta = \frac{\sigma L_e}{E_s\left(\frac{d}{2} + e - v\right)} \tag{10}$$

where $E_s = \sigma/\epsilon$ = stress/strain, for any point on the stress-strain diagram of the anchor bolt material in the elastic or plastic ranges, and L_e is the equivalent total length of the anchor bolts. By L_e is meant the length of a similar bolt, not embedded but fixed at the end only, that would have the same extension under a stress σ.

For use in similar anchorages where all linear dimensions are proportional to d, Equation 10 can be written eliminating the dimension d. Since in similar anchorages L_1 is proportional to d, and Appendix 1 shows L_2 likely to be proportional to d also; then, set L_e = the sum of L_1 and $L = L_e' d$. From earlier definition

$$e' = \frac{2e}{d}$$

and

$$v' = \frac{2v}{d}$$

then

$$\theta = \frac{2\sigma L_e'}{E_s(1 + e' - v')} \tag{11}$$

The above considerations on the M-θ relationship can be summarized as follows: If M' were plotted against θ, then the first part of the graph would be a straight line of approximate slope, $(c/6) \div (2c/E_c) = E_c/12$. From there on the Equations 9 and 10 with σ as parameter determine the shape of the curve.

It was pointed out earlier that the values v' and w', the equivalent anchor bolt length L_e, as well as the stress-strain relationship of the anchor bolts, must be known before the M'-θ curve can be drawn. Sufficient basic test information exists to make possible a fairly accurate estimate of the stress-strain relationship of the anchor bolts. An idealized curve is shown in Fig. 18, drawn as far as the maximum stress value. Alternatively, it would be relatively easy to actually test anchor bolts of a type proposed in any new design.

The estimate of L_e, the sum of L_1 and L_2, is somewhat more difficult. However, it is possible to proceed as follows: Gilkey(4) suggests that for a long embedded steel bar being pulled out of concrete bond is not effective over the whole length of the bar, but that it is effective only over a length of about 24 diameters. Assuming that this is so, the tensile stress in the bar decreases from σ at the point where it emerges from the concrete to zero at 24D (D = diameter of bar). It is shown in Appendix 1 that the length of bar L_2, which under a stress σ would strain by an amount equal to the slip s of the embedded bar, is equal to the distance of the center of gravity of the bond

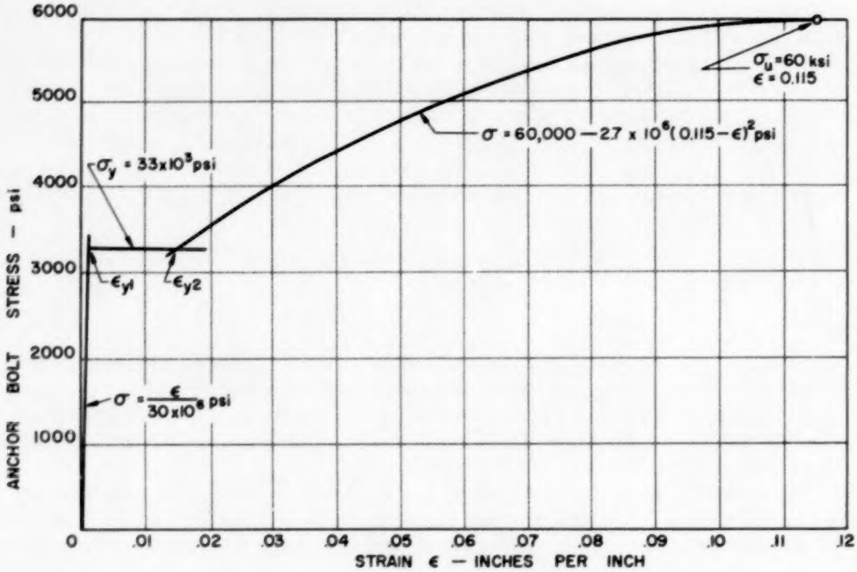

Fig. 18.
Idealized Stress-Strain Curve
For Anchor Bolts.

stress diagram from the concrete edge at the loaded end. Hence, to be able to estimate L_2 the bond stress distribution must be assumed. Assuming a linear decrease of bond stress from a value u_o at the loaded end to zero at 24 D,

$$L_2 = 1/3 \times 24D = 8D$$

and

$$L_e = L_1 + 8D .$$

The length L_1 of anchor bolt above the concrete surface is known and so is D.

If v and w (Fig. 17) are known at the various stages, the M'-θ diagram can be drawn. When this is done an M'-θ curve such as the one in Fig. 19 would result for a particular combination of values of p, c, $\alpha f_c'$, e, L, and a given anchor bolt stress-strain diagram. The initial slope of the curve has been estimated to be $E_c/12$. The ultimate values of M' and θ estimated earlier in this chapter were based on the assumption of rectangular bearing stress distribution. Hence, the coordinates of the terminal point of the curve can be estimated and the slope of the curve at that point would be zero. To fill in the rest of the curve, the bearing stress distribution must be estimated at intermediate stages, i.e., the values of v and w must be estimated. For this purpose it is considered justifiable to make use of the reinforced concrete theory for the stresses at the end of stage 3 (yield point reached) and a modified theory for the stresses at the end of stage 4 (strain-hardening range reached). (The modification consists of assuming trapezoidal stress distribution at the end of stage 4, with the maximum bearing stress limited to $\alpha f_c'$.)

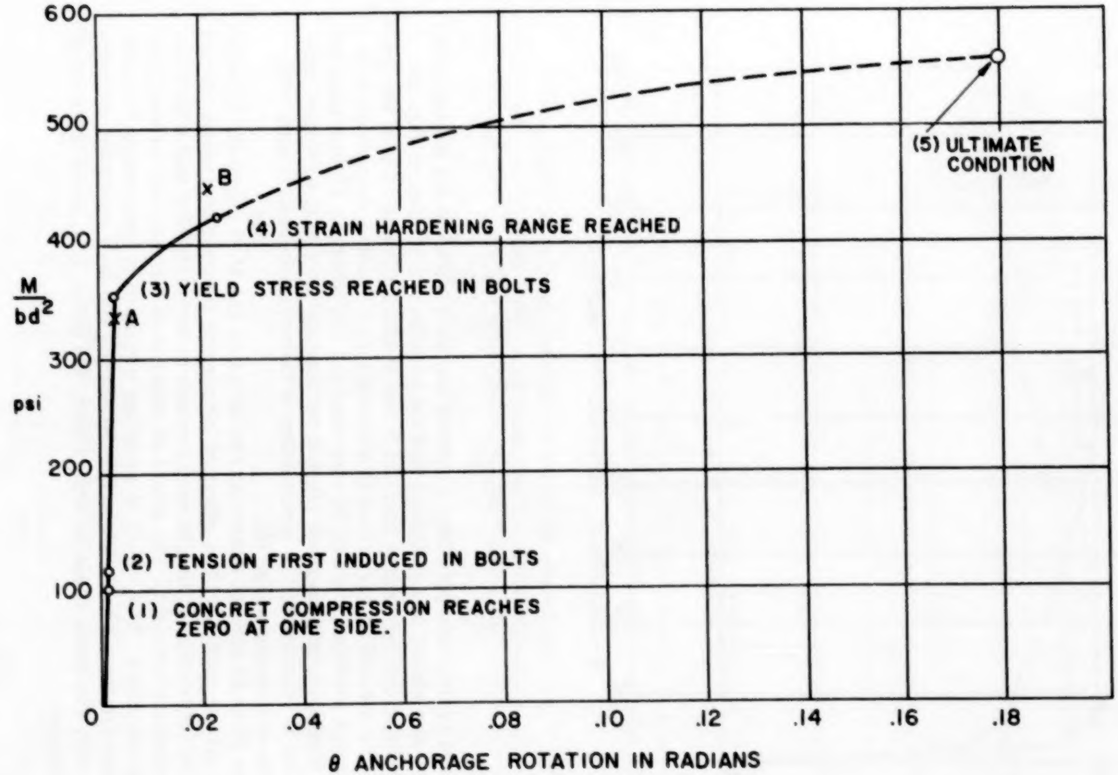

Fig. 19 Typical Theoretical Moment-Rotation Curve For Simple Column Anchorage
$p = 0.6\%$; $c = 600$ psi; $\sigma_y = 33000$ psi; $\sigma_u = 60000$ psi;
$\alpha f'_c = 6000$ psi; $e = .4d$; $L_1 = 2d/3$.

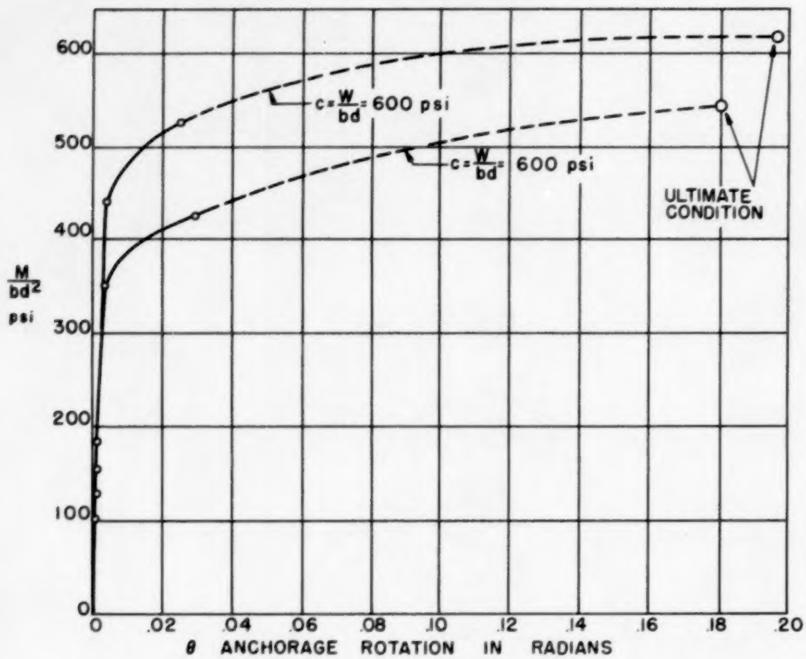

Fig. 20 The Effect of Different Initial Bearing Stresses, For $p = 0.6\%$; $\sigma_y = 33000$ psi; $\sigma_u = 60000$ psi; $\alpha f'_c = 6000$ psi; $e = .4d$; $L_1 = 2d/3$.

Details of typical computations to obtain the M-θ curve in the manner outlined above are shown in Appendix 2 and the results are plotted in Fig. 19. In Fig. 19 there are two points marked A and B which give an idea of the sensitivity of the method to variations in assumptions. Point A corresponds to point (3), but a modular ratio of 10 was used for (3), whereas 15 was used for A. Point B corresponds to point (4), but rectangular stress distribution was assumed for B (similar to the distribution at failure), whereas trapezoidal distribution was assumed for (4).

Figures 20 to 23 further illustrate the effect on the M-θ curve of the variation of c, $\alpha f'_c$, e, and L_1. The effect of increasing p would be similar to that produced by increasing c, i.e., it would result in more rigidity and a high ultimate resistance moment. The effect of changes in the concrete strength is not very pronounced. The result of reducing e to zero is interesting because such a procedure could be followed for estimating the M-θ curve for anchorages with only two bolts about an axis joining the bolts, i.e., about the axis of least resistance.

CONCLUSION

The foregoing procedure for estimating the moment-rotation characteristics of a simple column anchorage should be satisfactory as a first approximation. Experimental verification and subsequent correction of the

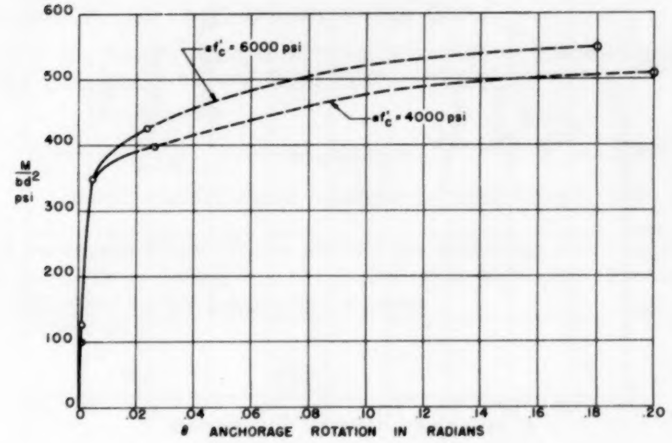

Fig. 21 The Effect of Variations in the Concrete Strength.
$p = 0.6\%$; $c = 600$ psi; $\sigma_y = 33000$ psi; $\sigma_u = 60000$ psi; $e = .4d$; $L_1 = 2d/3$.

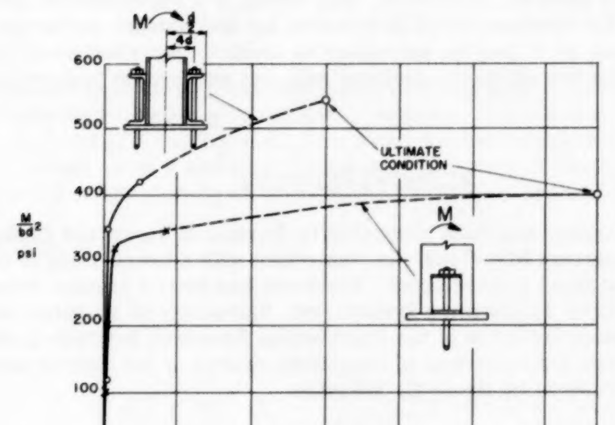

Fig. 22 The Effect of Varying the Eccentricity of the Anchor Bolts.
$p = 0.6\%$; $c = 600$ psi; $\sigma_y = 33000$ psi; $\sigma_u = 60000$ psi; $\alpha f'_c = 6000$ psi; $L_1 = 2d/3$.

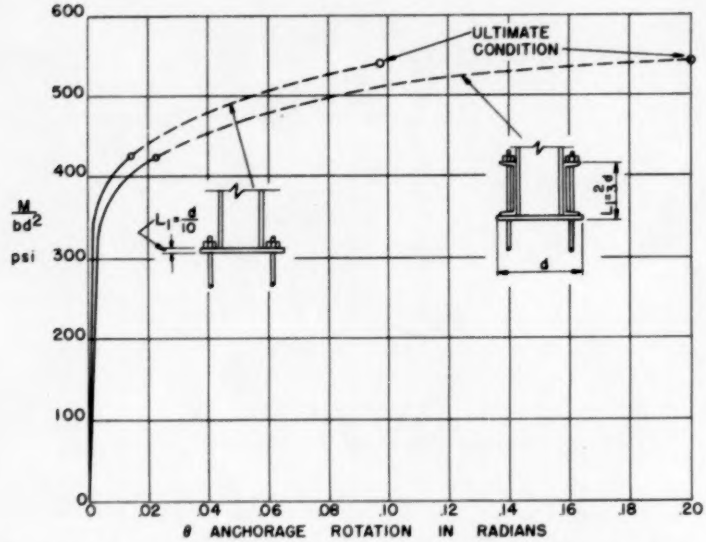

Fig. 23 The Effect of Varying the Free Anchor Bolt Length L_1.
$p = 0.6\%$; $c = 600$ psi; $\sigma_y = 33000$ psi; $\sigma_u = 60000$ psi; $\alpha f'_c = 6000$ psi; $e = 0.4d$.

assumptions should be made. Such a correction of the assumptions, after an initial series of tests on anchorages, may result in a refinement of the method permitting the estimate of the M-θ curve for any similar anchorage without further tests, or it may be necessary at times to carry out small supplementary tests in the nature of combined pull-out and tension tests on anchor bolts.

ACKNOWLEDGMENT

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BIBLIOGRAPHY

1. Timoshenko, S., Goodier, J. N., "Theory of Elasticity", McGraw-Hill Book Co., Inc., New York, 1951.

2. Graf, O., "Die Eigenschaften des Betons", Springer Verlag, Berlin (1950).

3. AISC, "Steel Construction", AISC, New York (1948).

4. Gilkey, H. J., Chamberlin, S. J., Beal, R. W., "Bond between concrete and steel", Iowa Eng. Exp. Sta. Bul. 147 (1940).

5. Newmark, N. M., "Analysis and design of structures subjected to dynamic loading". Proceedings, MIT Conference on "Building in the atomic age" (June 1952).

6. Brown, A. A., "Analysis of anchor bolts and concrete piers for large stills and kettles". Civ. Eng. 22:7:491-2 (July 1952).

7. Cox, H. L., "Static strength of rivets subjected to combined tension and shear". Master's Thesis, Univ. of Ill., (1952).

APPENDIX 1

EQUIVALENT FREE LENGTH OF EMBEDDED BAR

It is necessary to investigate whether the equivalent free length of an embedded bar is proportional to d, the length of the base plate, for bars of various sizes provided that p and o is the same.

If p is the same,

$$\frac{A_t}{bd} = \frac{\pi D^2}{4bd} = \text{constant},$$

where D = diameter of bar. Also, if the ratio of b to d is kept the same,

$$b = kd$$

$$\therefore \frac{D^2}{d^2} = \text{constant}$$

and it remains to be proven that the equivalent length is proportional to D.

The equivalent length L_2 is defined as follows: if the slip is s, then $s = P\,L_2/E_sA_t = \sigma L_2/E_s$ or $L_2 = sE_s/\sigma$. Since we are comparing various cases at a stage when θ and therefore σ are the same, σ can be considered a constant. Further, for L_2 to be proportional to D, s must be proportional to D.

From Fig. 24

$$P_x = P - \int_0^x u_x \pi D\, dx$$

$$\sigma_x = \sigma - \int_0^x \frac{4u_x}{D}\, dx \quad .$$

Hence, σ_x decreases as x increases and eventually becomes zero at L_t. The part of the bolt at a distance greater than L_t is of no importance.

$$0 = \sigma - \int_0^{L_t} \frac{4u_x}{D}\, dx, \text{ or } \sigma = \int_0^{L_t} \frac{4u_x}{D}\, dx.$$

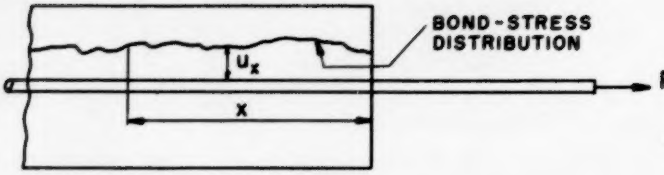

Fig. 24

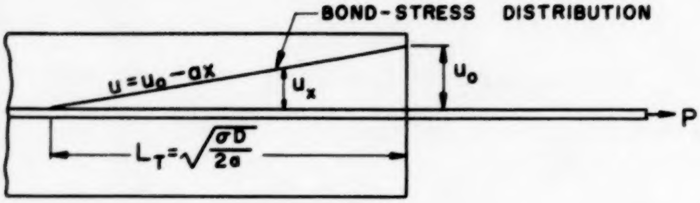

Fig. 25.

The strain $\epsilon_x = \frac{1}{E_s} \sigma_x$

$$\therefore E_s \; \epsilon_x = \int_0^{L_t} \frac{4u_x}{D} dx - \int_0^{x} \frac{4u_x}{D} dx = \int_x^{L_t} \frac{4u_x}{D} dx$$

$$\therefore s = \int_0^{L_t} \epsilon_x \, dx = \frac{4}{DE_s} \int_0^{L_t} \int_x^{L_t} u_x \, dx^2 \quad .$$

(i) If u_x is constant and independent of x

$$s = \frac{2uL_t^2}{DE_s} \quad .$$

It has been shown by Gilkey (5.4) and others that the effective length L_t is about 24D. But in any case L_t can at least be assumed to be proportional to D. Hence,

$$u = \frac{\sigma \; D}{4 \; L_t} = \frac{\sigma}{4K} \quad ,$$

where K is a constant and

$$s = \frac{2uL_t^2}{DE_s} = \frac{2\sigma \; K^2 D^2}{4K \; DE_s} = \frac{\sigma KD}{2E_s} \quad ,$$

i.e., for any one σ, the slip is proportional to the diameter. Furthermore, the equivalent length L_2 is found to be

$$\frac{E_s}{\sigma} = \frac{KD}{2} = \frac{L_t}{2} \quad .$$

This means that the equivalent free length L_2 is half the effective length L_t, i.e., the amount of slip at the loaded end is the same as the extension of a nonembedded bar fixed at the center of gravity of the bond-stress distribution diagram.

(ii) If the bond stress varies linearly, as shown in Fig. 25, then

$$s = \frac{2u_oL_t^2}{3\ DE_s} \quad \text{and} \quad \sigma = \frac{2u_oL_t}{D} = 2u_oK$$

$$= \frac{\sigma\ KD}{3\ E_s}, \text{ i.e., } s \text{ is proportional to } D$$

and

$$L_2 = \frac{1}{3} L_t$$

It would appear to be generally true that $s \propto D \propto d$ for the same p and σ.

APPENDIX 2

TYPICAL COMPUTATIONS FOR MOMENT-ROTATION CURVE

Data: c = 600 psi; p = 0.6%; $\alpha f_c' = 6000$ psi; e = 0.4d;
$L_1 = 2d/3$; stress-strain curve for anchor bolt material as in Fig. 18;
n for concrete is 10.

By definition

$$e' = \frac{2e}{d} = 0.8; \qquad l_1' = \frac{L_1}{d} = \frac{2}{3} \quad .$$

Also

$$p = \frac{A_t}{bd} = 0.006, \text{ therefore, } \frac{\pi D^2}{4} = 0.006bd \ .$$

Assuming

$$b \approx d, \ D = d\sqrt{\frac{0.012}{\pi}} = 0.062d$$

Further assuming a bond-stress distribution as in Fig. 25 (Appendix 1) and a total effective length of embedment of 24D, the equivalent length L_2 of the embedded portion of the bolt is 8D. Thus, L_2 = 8D = 0.496d. The total

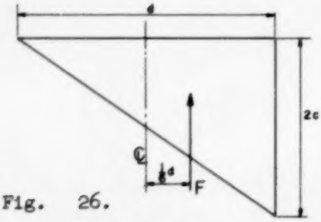

Fig. 26.

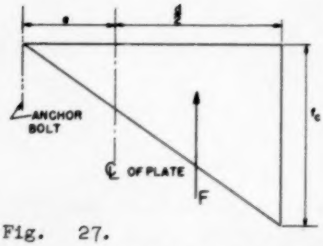

Fig. 27.

equivalent length of the anchor bolt $L_e = L_1 + L_2 = (0.667 + 0.496)d = 1.163d.$
By definition

$$L'_e = \frac{L_e}{d} = 1.163.$$

a. Stage 1

At the end of stage 1, as defined on page 15, the stress distribution under the plate is as shown in Fig. 26. The moment of resistance is

$$M = \frac{1}{6}dF = \frac{bd^2c}{6} \quad .$$

By definition

$$M' = \frac{M}{bd^2} = \frac{c}{6} \quad ,$$

Therefore

$$\underline{M'_1 = 100 \text{ psi}} \quad .$$

Similarly the rotation at the end of stage 1 is given by

$$\theta_1 = \frac{2c}{E_c} \qquad \underline{\theta_1 = 4\text{x}10^{-4} \text{ rad}} \, .$$

b. Stage 2

At the end of the second stage, tensile stress is just being induced in the anchor bolt on one side. The stress distribution under the plate is as shown in Fig. 27. The reaction is

$$F = \frac{1}{2} f_c b \left(\frac{d}{2} + e\right) = \frac{f_c bd}{4} (1 + e').$$

But since there has been no additional stress induced in the anchor bolt as yet, F must also equal the original reaction bdc. Thus,

$$bdc = \frac{f_c bd}{4} (1 + e')$$

and

$$f_c = \frac{4c}{1 + e'} \quad .$$

The moment of resistance is given by

$$M_2 = bdc\left[\frac{d}{2} - \frac{1}{3}\left(\frac{d}{2} + e\right)\right] = \frac{bd^2c}{2}\left[1 - \frac{1}{3}(1 + e')\right]$$

and

$$M_2' = \frac{c}{6}(2 - e') = 100\ (2 - 0.8);$$

$$\underline{M_2' = 120 \text{ psi}} \ .$$

The strain in the concrete due to a stress f_c is given by

$$\frac{f_c}{E_c} = \frac{4c}{(1+e')E_c} \quad .$$

Assuming, as indicated in the text, that an effective depth d strains this amount

$$\theta_2 = \frac{4cd}{(1+e')\ E_c} \cdot \frac{1}{e+\frac{d}{2}} = \frac{8c}{(1+e')^2 E_c} \ ;$$

$$\underline{\theta_2 = 5 \times 10^{-4} \text{ rad.}}$$

c. Stage 3

During this stage, the anchor plate is tilted sufficiently so that it is in contact with the concrete over a strip of width v. At the end of the stage (by definition) the stress in the anchor bolt has reached the yield point (Fig. 28).

To determine v and w, the following two equations can be written and solved. By geometry

$$\frac{\sigma_y}{f_c} = \frac{n(\frac{d}{2} + e - v)}{v} = \frac{n(1 + e' - v')}{v'} \quad .$$

From equilibrium $pbd\sigma_y + bdc = \frac{vbf_c}{2}$.

Thus,

$$\frac{\sigma_y\ v'^2}{4n(p\sigma_y + c)} + v' + (1 + e') = 0$$

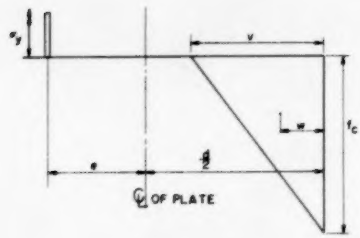

Fig. 28.

from which

$$v' = 0.92; \quad w' = \frac{1}{3}v' = 0.307 \quad .$$

From Equation 9,

$$M' = \frac{1}{2} \times 798\,(1-0.307) = \frac{1}{2} \times 198 \times 0.8 \quad ,$$

therefore, $\underline{M_3' = 354 \text{ psi}}$.

Using Equation 11

$$\theta_3 = \theta_2 + \frac{2 \times 33 \times 1.163}{30 \times 10^3\,(1.8-0.92)} \; ; \quad \underline{\theta_3 = 34 \times 10^{-4} \text{ rad.}}$$

d. Stage 4

This stage ends when the strain in the anchor bolts reaches the value ϵ_{y2} (Fig. 18). At this time the concrete stress distribution is nonlinear and may be approximated by a trapezoid (Fig. 29). To determine v and y two equations can be written and solved. By geometry

$$\frac{\epsilon_{y2}}{\alpha f_c'/E_c} = \frac{\frac{d}{2} + e - v}{v-y}$$

From equilibrium $pbd\sigma_y + bdc = \frac{1}{2}\alpha f_c'\,(v + y)b$

from which

$$v' = \frac{2v}{d} = 0.353 \qquad \text{and } y' = \frac{2y}{d} = 0.178 \; .$$

Also,

$$w' = \frac{v'^2 + v'y' + y'^2}{3(v' + y')} = 0.137 \quad .$$

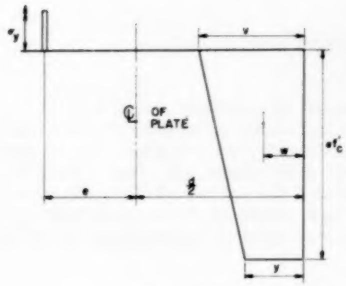

Fig. 29.

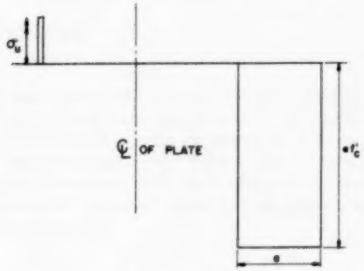

Fig. 30.

As in stage 3, $M' = \frac{1}{2} \times 798\,(1-0.137) + \frac{1}{2} \times 0.198 \times 0.8$

$$\underline{M_4' = 423 \text{ psi}} .$$

Using Equation 11, but replacing σ_y/E_s by ϵ_{y2} ,

$$\theta_4 = \theta_2 + \frac{2\epsilon_{y2} L_e'}{1+e' - v'} \quad ; \quad \underline{\theta_4 = 246 \times 10^{-4} \text{ rad.}}$$

e. Stage 5

This stage represents the condition of ultimate moment. From Equation 5a

$$a = \frac{(c + p\sigma_u)d}{\alpha f_c'} = 0.16d$$

$$M_u' = \frac{1}{2} \times 960(1-0.16) + \frac{1}{2} \times 360 \times 0.8$$

$$\underline{M_u' = 547 \text{ psi}}$$

$$\theta_5 = \theta_2 + \frac{\epsilon_u L_e}{\frac{d}{2}+e - a} = \theta_2 + \frac{2\epsilon_u L_e'}{1+e'-a'}$$

$$\underline{\theta_5 = 0.18 \text{ rad.}}$$

The above values of M' and θ are shown in graph form in Fig. 19.

PROCEEDINGS-SEPARATES

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APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

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JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.